



INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI  
DEPARTMENT OF MATHEMATICS AND STATISTICS

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## Syllabus for PhD Admission Written Test

### August 2023 Notifications

### Statistics

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**Written Test:** Duration of the written test is 2 hours.

**Interview:** The interview will be mainly based on three areas chosen by the candidate, typically their area(s) of interest.

## Syllabus

Candidates are required to answer questions from **at least three** of the following 4 general areas.

### 1.1 Linear Algebra

System of Linear Equations, Matrices and Elementary Row Operations, Row-Reduced Echelon Matrices. Vector Spaces, Subspaces, Bases and Dimension, Ordered Basis and Coordinates. Linear Transformations, Rank-Nullity Theorem, The Algebra of Linear Transformations, Isomorphism, Matrix Representation of Linear Transformations, Linear Functionals, Annihilator, Double Dual, Transpose of a Linear Transformation. Characteristic Values and Characteristic Vectors of Linear Transformations, Diagonalizability, Minimal Polynomial of a Linear Transformation, Cayley-Hamilton Theorem, Invariant Subspaces, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem, Cyclic Subspaces and Annihilators, Cyclic Decomposition, Rational, Jordan Forms. Inner Product Spaces, Orthonormal Basis, Gram-Schmidt Theorem.

### 1.2 Real Analysis

**Real Analysis:** Real Number System and its Completeness, Sequences and Series of Real Numbers. Metric Spaces: Basic Concepts, Continuous Functions, Completeness, Contraction Mapping Theorem, Connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem. Differentiation, Taylor's Theorem, Riemann Integral, Improper Integrals. Sequences and Series of Functions, Uniform Convergence, Power Series, Fourier Series, Weierstrass Approximation Theorem, Equicontinuity, Arzela-Ascoli Theorem.

### **1.3 Probability Theory**

Probability Measure, Probability Space, Limit of Events, Borel-Cantelli Lemma, Random Variables, Random Vectors. Distributions, Multi-dimensional Distributions, Independence, Expectation, Change of Variable Theorem, Moment Generating Function, and Characteristics Functions, Inversion and Uniqueness Theorems. Sequences of Random Variables, Modes of Convergence, Weak and Strong Laws of Large Numbers, Central Limit Theorem. Definition, Properties of Conditional Expectation, and Conditional Probability.

### **1.4 Statistics**

Concept of Statistical Inference, Point Estimation, Methods of Estimations, Properties of Estimation, Uniformly Minimum Variance Unbiased Estimators (UMVUE), Rao-Cramer Lower Bound, Bhattacharya's Bound, Minimal Sufficiency, Rao-Blackwell Theorem, Lehmann-Scheffe Theorem, Order Statistics, Interval Estimation. Testing of Hypothesis, Type-I and II error, Power of the test, The Neyman-Pearson Fundamental Lemma, Uniformly Most Powerful Test, Unbiased Test, Invariance, Likelihood Ratio Test, Decision Theory, Bayes and Minimax Procedure, Minimum Risk Equivariant (MRE) Estimators.