Syllabus for PhD Admission Written Test
August 2023 Notifications
Mathematics

Written Test: Duration of the written test is 2 hours.
Interview: The interview will be mainly based on three areas chosen by the candidate, typically their area(s) of interest.

Syllabus

Candidates are required to answer questions from **at least three** of the following **7** general areas related to their **area of interest**.

1.1 Linear Algebra

System of Linear Equations, Matrices and Elementary Row Operations, Row-Reduced Echelon Matrices. Vector Spaces, Subspaces, Bases and Dimension, Ordered Basis and Coordinates. Linear Transformations, Rank-Nullity Theorem, The Algebra of Linear Transformations, Isomorphism, Matrix Representation of Linear Transformations, Linear Functionals, Annihilator, Double Dual, Transpose of a Linear Transformation. Characteristic Values and Characteristic Vectors of Linear Transformations, Diagonalizability, Minimal Polynomial of a Linear Transformation, Cayley-Hamilton Theorem, Invariant Subspaces, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem, Cyclic Subspaces and Annihilators, Cyclic Decomposition, Rational, Jordan Forms. Inner Product Spaces, Orthonormal Basis, Gram-Schmidt Theorem.

1.2 Abstract Algebra

Basic Group Theory, Normal Subgroups, Quotient Groups and Homomorpism Theorems, Group Actions with examples, Cayley's Theorem, The Class Equation and their application, Sylow's Theorems, Direct Products, Structure Theorem for Finite Abelian Groups, Existence and universal Properties of free Groups. Basic Ring Theory, Commutative Ring With Identity, Properties of Ideals, Prime and Maximal Ideals, Zorn's lemma and existence of maximal ideals, Quotient Rings and Localization, Chinese Reminder Theorem, Field of Fractions and

Integral Domains, Euclidean Domain, Principal Ideal Domain(PID), Unique Factorization Domain(UFD), Irreducibility Criterion, Primes in Z[i] and Fermat's Two-Square Theorem, Definition and simple examples of modules over commutative and non-commutative rings. Field Theory: Finite, Algebraic and Transcendental Extensions, Existence and Cardinality of Algebraic Closure, Splitting fields and Normal extensions, Galois Theory of Polynomial in characteristic zero and simple examples, Separable, Inseparable and Purely inseparable extensions, Finite Fields.

1.3   Real Analysis and Complex Analysis

Real Analysis: Real Number System and its Completeness, Sequences and Series of Real Numbers. Metric Spaces: Basic Concepts, Continuous Functions, Completeness, Contraction Mapping Theorem, Connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem. Differentiation, Taylor's Theorem, Riemann Integral, Improper Integrals. Sequences and Series of Functions, Uniform Convergence, Power Series, Fourier Series, Weierstrass Approximation Theorem, Eqicontinuity, Arzela-Ascoli Theorem.

Complex Analysis: Topology of the Complex Plane, Riemann Sphere, Limits, Continuity and Differentiability, Analytic Functions, Harmonic Functions and Multi-Valued Functions. Convergence of Numerical Series, Radius of Convergence of Power Series and Power Series as an Analytic Function, Laurent Series. Cauchy's Integral Theorem, Cauchy Integral Formula, Morera's Theorem, Taylor's Theorem, Laurent's Theorem, Liouville's Theorem, Schwarz Lemma, Maximum Modulus Principle. Conformal Mappings, Linear Fractional Transformations, Classification of Singularities, Cauchy's Residue Theory and Evaluation of Real Integrals.

1.4   Functional Analysis

Normed Linear Space, Banach Spaces and Basic Properties: Heine-Borel Theorem, Riesz Lemma and Best Approximation Property, Inner Product Space and Projection Theorem, Orthonormal Bases, Bessel Inequality and Parseval's Formula, Riesz-Fischer Theorem. Bounded Operators and Basic Properties, Space of Bounded Operators and Dual Space, Riesz Representation Theorem, Adjoint of Operators on a Hilbert Space, Examples of Unbounded Operators, Convergence of Sequence of Operators. Hahn-Banach Extension Theorem, Uniform Boundedness Principle, Closed Graph Theorem and Open Mapping Theorem, Some Applications. Invertibility of Operators, Spectrum of an Operator.

1.5   Differential Equations

Ordinary Differential Equations: Existence-Uniqueness: Review of Exact Equations of First Order, The Method of Successive Approximations, Lipschitz Condition, Convergence of Successive Approximations, Existence and Uniqueness of Solutions for First Order Initial Value Problem, Non-Local Existence of Solutions, Existence and Uniqueness of Solutions to Systems, Existence and Uniqueness for Linear Systems, Equations of Order n. Second Order Equations: General Solution of Homogeneous Equations, Non-Homogeneous Equations, Wronskian,

Method of Variation of Parameters, Sturm Comparison Theorem, Sturm Separation Theorem, Boundary Value Problems, Green's Functions, Sturm-Liouville Problems. Series of Solution of Second Order Linear Equations: Ordinary Points, Regular Singular Points, Legendre Polynomials and Properties, Bessel Functions and Properties. Systems of Differential Equations: Algebraic Properties of Solutions of Linear Systems, The Eigenvalue-Eigenvector Method of Finding Solutions, Complex Eigenvalues, Equal Eigenvalues, Fundamental Matrix Solutions, Matrix Exponential, Nonhomogeneous Equations, Variation of Parameters.

Partial Differential Equations: Linear PDEs, First Linear and Quasi-linaer PDEs, Classification of Second Order PDEs, Cauchy Problem, Variable Separable, Wave Equation, Heat Equation, Laplace Equation, Transport Equation, D'Alembert's Principle, Boundary Value Problems, Green's Function

1.6 Numerical Analysis

Norms of Vectors and Matrices, Linear Systems: Direct and Iterative Schemes, Ill-Conditioning and Convergence Analysis. Numerical Schemes for Non-linear Systems, Regression. Numerical Solution of Differential Equations: Single Step and Multi-Step Methods, Order, Consistency, Stability and Convergence Analysis, Stiff Equations. Two Point Boundary Value Problems, Shooting and Finite Difference Methods.

1.7 Topology

Topological Spaces, Basis for a Topology, Subspace Topology, Closed Sets and Limit Points, Continuous Functions, Product Topology, Quotient Topology.

Connected Spaces, Connected Subspaces of the Real Line, Components and Local Connectedness, Path Connectedness, Compact Spaces, Limit Point Compactness, Local Compactness.

The Countability and Separation Axioms, The Urysohn Lemma, The Urysohn Metrization Theorem, The Tietze Extension Theorem, Tychonoff Theorem.