



INDIAN INSTITUTE OF TECHNOLOGY TIRUPATI  
DEPARTMENT OF MATHEMATICS AND STATISTICS

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## Syllabus for PhD Admission Written Test July 2022

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**Written Test:** Duration of the written test is 2 hours.

**Interview:** The interview will be conducted for the candidates who cleared their written test. The interview will be mainly based on three areas chosen by the candidate, typically their area(s) of interest.

### Syllabus

Candidates are required to answer questions from **at least three** of the following 7 general areas related to their **area of interest**.

#### 1.1 Linear Algebra

System of Linear Equations, Matrices and Elementary Row Operations, Row-Reduced Echelon Matrices. Vector Spaces, Subspaces, Bases and Dimension, Ordered Basis and Coordinates. Linear Transformations, Rank-Nullity Theorem, The Algebra of Linear Transformations, Isomorphism, Matrix Representation of Linear Transformations, Linear Functionals, Annihilator, Double Dual, Transpose of a Linear Transformation. Characteristic Values and Characteristic Vectors of Linear Transformations, Diagonalizability, Minimal Polynomial of a Linear Transformation, Cayley-Hamilton Theorem, Invariant Subspaces, Direct-Sum Decompositions, Invariant Direct Sums, The Primary Decomposition Theorem, Cyclic Subspaces and Annihilators, Cyclic Decomposition, Rational, Jordan Forms. Inner Product Spaces, Orthonormal Basis, Gram-Schmidt Theorem.

#### 1.2 Real Analysis and Complex Analysis

**Real Analysis:** Real Number System and its Completeness, Sequences and Series of Real Numbers. Metric Spaces: Basic Concepts, Continuous Functions, Completeness, Contraction Mapping Theorem, Connectedness, Intermediate Value Theorem, Compactness, Heine-Borel Theorem. Differentiation, Taylor's Theorem, Riemann Integral, Improper Integrals. Sequences and Series of Functions, Uniform Convergence, Power Series, Fourier Series, Weierstrass Approximation Theorem, Equicontinuity, Arzela-Ascoli Theorem.

**Complex Analysis:** Topology of the Complex Plane, Riemann Sphere, Limits, Continuity and Differentiability, Analytic Functions, Harmonic Functions and Multi-Valued Functions. Convergence of Numerical Series, Radius of Convergence of Power Series and Power Series as an Analytic Function, Laurent Series. Cauchy's Integral Theorem, Cauchy Integral Formula, Morera's Theorem, Taylor's Theorem, Laurent's Theorem, Liouville's Theorem, Schwarz Lemma, Maximum Modulus Principle. Conformal Mappings, Linear Fractional Transformations, Classification of Singularities, Cauchy's Residue Theory and Evaluation of Real Integrals.

### 1.3 Functional Analysis

Normed Linear Space, Banach Spaces and Basic Properties: Heine-Borel Theorem, Riesz Lemma and Best Approximation Property, Inner Product Space and Projection Theorem, Orthonormal Bases, Bessel Inequality and Parseval's Formula, Riesz-Fischer Theorem. Bounded Operators and Basic Properties, Space of Bounded Operators and Dual Space, Riesz Representation Theorem, Adjoint of Operators on a Hilbert Space, Examples of Unbounded Operators, Convergence of Sequence of Operators. Hahn-Banach Extension Theorem, Uniform Boundedness Principle, Closed Graph Theorem and Open Mapping Theorem, Some Applications. Invertibility of Operators, Spectrum of an Operator.

### 1.4 Differential Equations

**Ordinary Differential Equations:** Existence-Uniqueness: Review of Exact Equations of First Order, The Method of Successive Approximations, Lipschitz Condition, Convergence of Successive Approximations, Existence and Uniqueness of Solutions for First Order Initial Value Problem, Non-Local Existence of Solutions, Existence and Uniqueness of Solutions to Systems, Existence and Uniqueness for Linear Systems, Equations of Order  $n$ . Second Order Equations: General Solution of Homogeneous Equations, Non-Homogeneous Equations, Wronskian, Method of Variation of Parameters, Sturm Comparison Theorem, Sturm Separation Theorem, Boundary Value Problems, Green's Functions, Sturm-Liouville Problems. Series of Solution of Second Order Linear Equations: Ordinary Points, Regular Singular Points, Legendre Polynomials and Properties, Bessel Functions and Properties. Systems of Differential Equations: Algebraic Properties of Solutions of Linear Systems, The Eigenvalue-Eigenvector Method of Finding Solutions, Complex Eigenvalues, Equal Eigenvalues, Fundamental Matrix Solutions, Matrix Exponential, Nonhomogeneous Equations, Variation of Parameters.

**Partial Differential Equations:** Linear PDEs, First Linear and Quasi-linear PDEs, Classification of Second Order PDEs, Cauchy Problem, Variable Separable, Wave Equation, Heat Equation, Laplace Equation, Transport Equation, D'Alembert's Principle, Boundary Value Problems, Green's Function

### 1.5 Numerical Analysis

Norms of Vectors and Matrices, Linear Systems: Direct and Iterative Schemes, Ill-Conditioning and Convergence Analysis. Numerical Schemes for Non-linear Systems, Regression. Numerical

Solution of Differential Equations: Single Step and Multi-Step Methods, Order, Consistency, Stability and Convergence Analysis, Stiff Equations. Two Point Boundary Value Problems, Shooting and Finite Difference Methods.

## **1.6 Probability Theory**

Probability Measure, Probability Space, Limit of Events, Borel-Cantelli Lemma, Random Variables, Random Vectors. Distributions, Multi-dimensional Distributions, Independence, Expectation, Change of Variable Theorem, Moment Generating Function, and Characteristics Functions, Inversion and Uniqueness Theorems. Sequences of Random Variables, Modes of Convergence, Weak and Strong Laws of Large Numbers, Central Limit Theorem. Definition, Properties of Conditional Expectation, and Conditional Probability.

## **1.7 Statistics**

Concept of Statistical Inference, Point Estimation, Methods of Estimations, Properties of Estimation, Uniformly Minimum Variance Unbiased Estimators (UMVUE), Rao-Cramer Lower Bound, Bhattacharya's Bound, Minimal Sufficiency, Rao-Blackwell Theorem, Lehmann-Scheffe Theorem, Order Statistics, Interval Estimation. Testing of Hypothesis, Type-I and II error, Power of the test, The Neyman-Pearson Fundamental Lemma, Uniformly Most Powerful Test, Unbiased Test, Invariance, Likelihood Ratio Test, Decision Theory, Bayes and Minimax Procedure, Minimum Risk Equivariant (MRE) Estimators.